

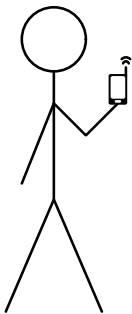
Practical proof systems:  
Implementations, applications, and next steps

Riad S. Wahby

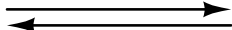
Stanford

September 23<sup>rd</sup>, 2019

Verifier  $\mathcal{V}$



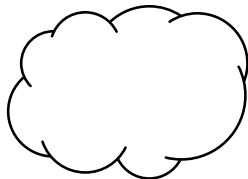
computation  $\Phi$ ,  
input  $x$

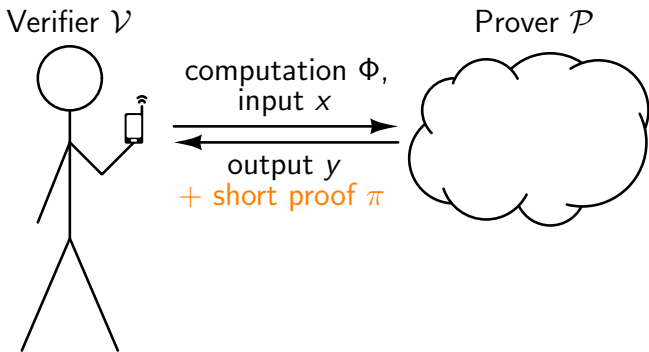


output  $y$

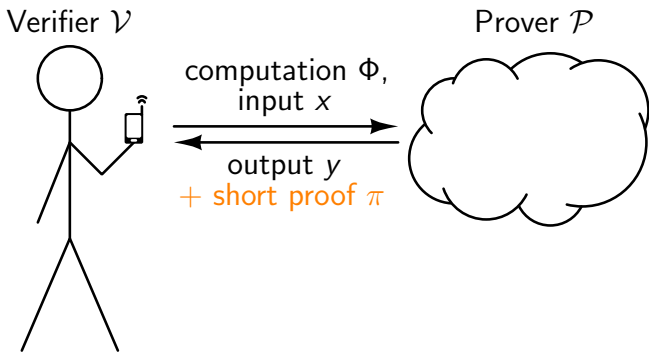


Prover  $\mathcal{P}$



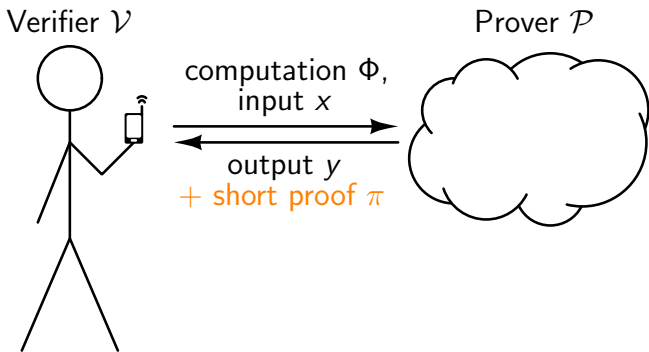


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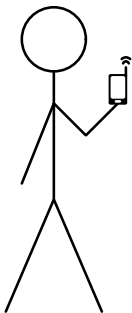
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SBW11  
CMT12  
SMBW12  
SVPBBW12  
TRMP12  
BCGTV13  
BFRSBW13  
BFR13  
DFKP13  
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FGP14  
FL14  
KPPSST14

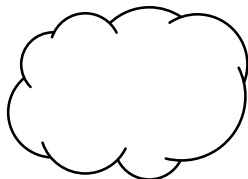
Verifier  $\mathcal{V}$



computation  $\Phi$ ,  
input  $x$

output  $y$   
+ short proof  $\pi$

Prover  $\mathcal{P}$



BBFR15  
CFHKKNPZ15  
CTV15  
KZMQCPPsS15  
WSRHBW15  
D-LFKP16  
FFGKOP16  
GMO16  
NT16  
WHGsW16  
AHIV17  
CDGORRSZ17  
WJBsTWW17  
ZGKPP17a  
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- model of computation / “expressiveness”

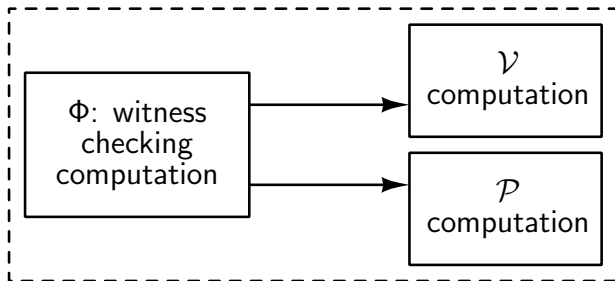
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**Bottom line:** this is a huge tradeoff space!

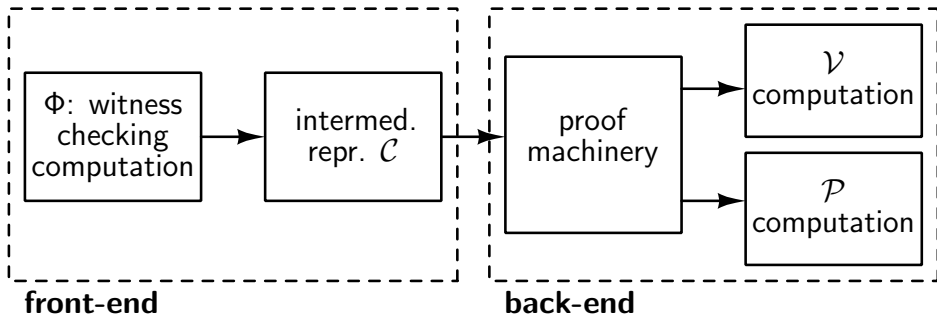
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$\Phi$ : witness  
checking  
computation

intermed.  
repr.  $\mathcal{C}$

proof  
machinery

$\mathcal{V}$   
computation

$\mathcal{P}$   
computation

**front-end**

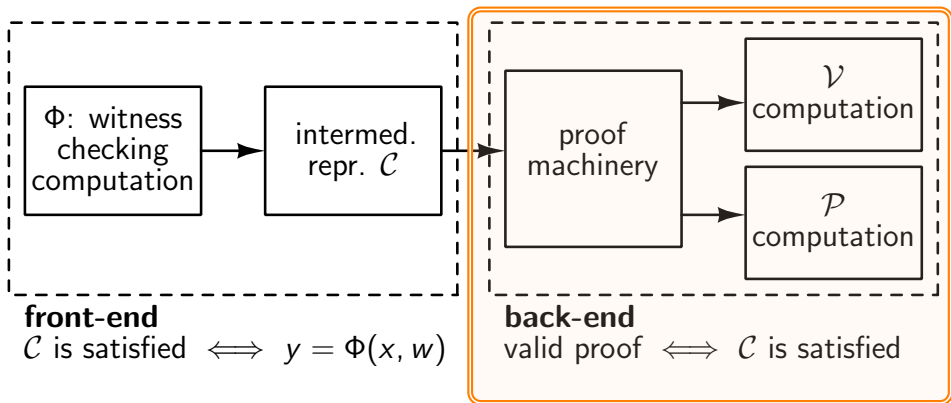
$\mathcal{C}$  is satisfied  $\iff y = \Phi(x, w)$

**back-end**



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## Underlying machinery

Linear PCPs [IKO07,BBCGI19] and QAPs [GGPR13]

Pepper [SBW11,SMBW12], Ginger [SVPBBW12], Zaatar [SBVBPW13],  
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(setup amortizes over a batch)

Pinocchio [PGHR13], libSNARK [BCTV14a],  
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(setup amortizes forever)

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ZKBoo [GMO16], ZKB++ [CDGORRSZ17], Ligerio [AHIV17],  
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[CMT12,Tha13], Clover [BTVW14], Giraffe [WJBsTWW17] (unconditional)

## Universal trusted setup

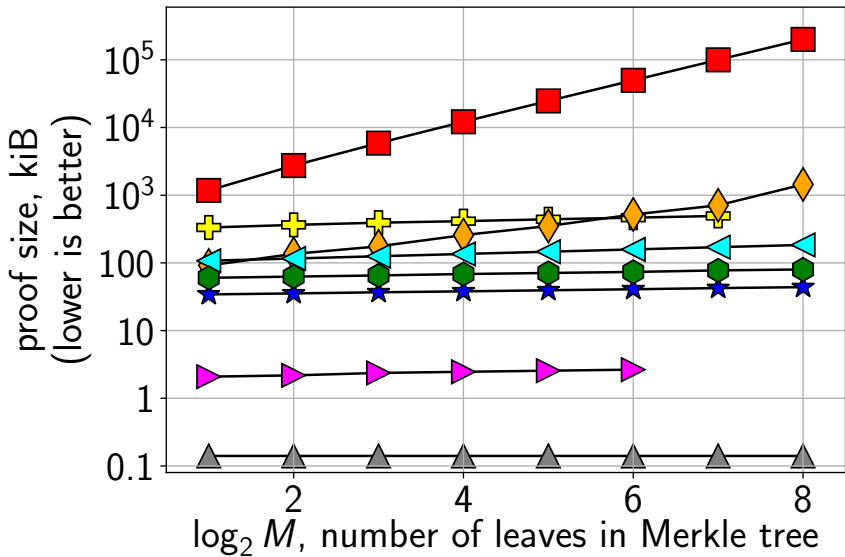
[BCGTV13], libSNARK/vnTinyRAM [BCTV14a], [BCTV14b],  
(ZK)vSQL [ZGKPP17{a,b}], vRAM [ZGKPP18], Libra [XZZPS19] (KoE)

Sonic [MBKM19] (algebraic group model)

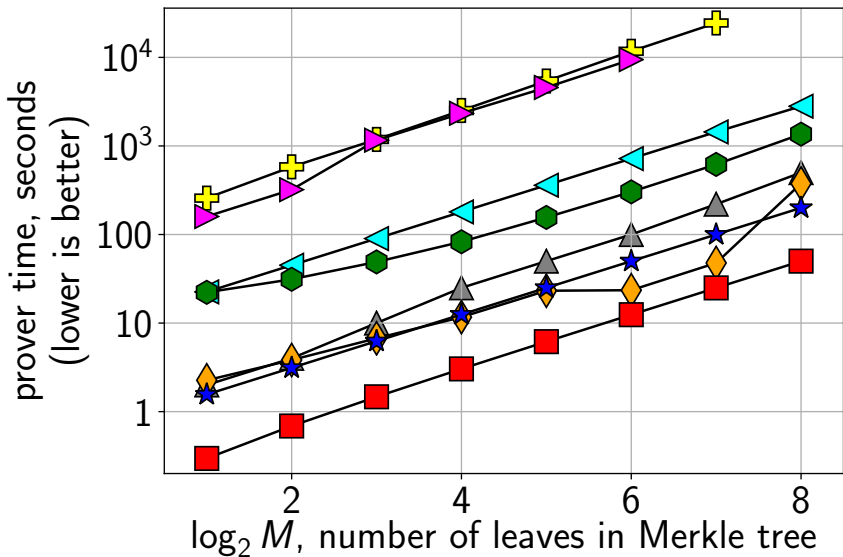
## Per- $\Phi$ trusted setup

Pepper, Ginger, Zaatar [SBW11,SMBW12,SVPBBW12,SBVBPW13] (DH)  
(setup amortizes over a batch)

Pinocchio [PGHR13], libSNARK [BCTV14a],  
ADSNARK [BBFR15], Geppetto [CFHKKNPZ15], ... (KoE)  
(setup amortizes forever)

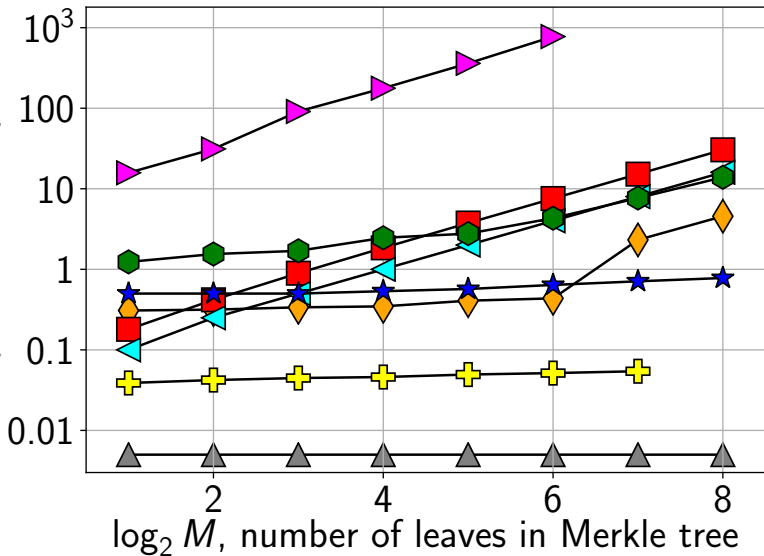


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verifier time, seconds  
(lower is better)



Numbers are from Hyrax [WTsTW18], except Libra, Aurora, and libSNARK, which are from Libra [XZZPS19]

## Proof system construction

On input  $x$ ,  $\mathcal{P}$  convinces  $\mathcal{V}$  that  $y = \Phi(x, w)$   
for a witness  $w$  that  $\mathcal{P}$  knows

$\Phi$ : witness  
checking  
computation

intermed.  
repr.  $\mathcal{C}$

proof  
machinery

$\mathcal{V}$   
computation

$\mathcal{P}$   
computation

**front-end**

$\mathcal{C}$  is satisfied  $\iff y = \Phi(x, w)$

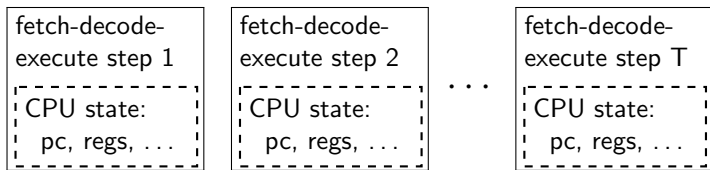
**back-end**

valid proof  $\iff \mathcal{C}$  is satisfied



## Representing $\Phi$ for execution on the back-end

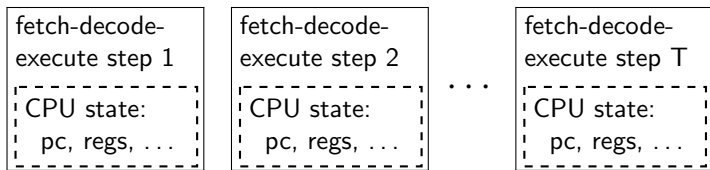
“CPU”: run  $\Phi$  on unrolled FSM



[BCGTV13,BCTV14a,BCTV14b,CTV15,ZGKPP18,BBHR19]

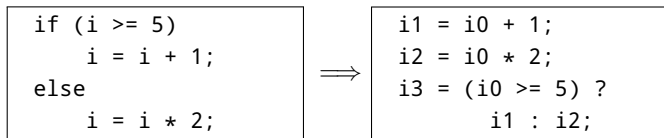
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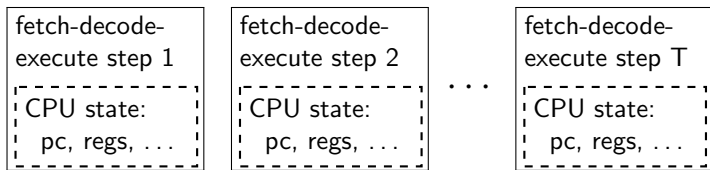
“FPGA”: translate  $\Phi$  directly to AC or constraints



[..., SVPBBW12, BFRSBW13, SBVBPW13, PGHR13, VSBW13, BBFR15, CFHKKNPZ15, KZMQCPPsS15, WSRHBW15, BCCGP16, BBBPWM18, KPS18, BCRSVW19, MBKM19, Circom, Bellman, ...]

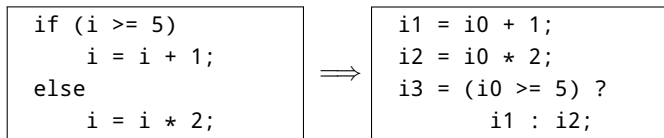
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[CMT12, WHGsW16, WJBsTWW17, WTstTW18, XZZPS19]

## Representing $\Phi$ for execution on the back-end

### “CPU”: run $\Phi$ on unrolled FSM

```
// assume i0 is k+1 bits
i4 = i0 - 5;
// prover supplies i4_0, ..., i4_k
assert (i4 - i4_0 - 2 * i4_1 - ... - 2^k * i4_k == 0);
assert (i4_0 * (1 - i4_0) == 0);
...
assert (i4_{k-1} * (1 - i4_{k-1}) == 0);
assert (i4_k == 0);
```

```
if (i >= 5)
    i = i + 1;
else
    i = i * 2;
```



```
i1 = i0 + 1;
i2 = i0 * 2;
i3 = (i0 >= 5) ?
    i1 : i2;
```

bits

[..., SVPBBW12, BFRSBW13, SBVBPW13, PGHR13, VSBW13, BBFR15, CFHKKNPZ15, KZMQCPPsS15, WSRHBW15, BCCGP16, BBBPWM18, KPS18, BCRSVW19, MBKM19, Circom, Bellman, ...]

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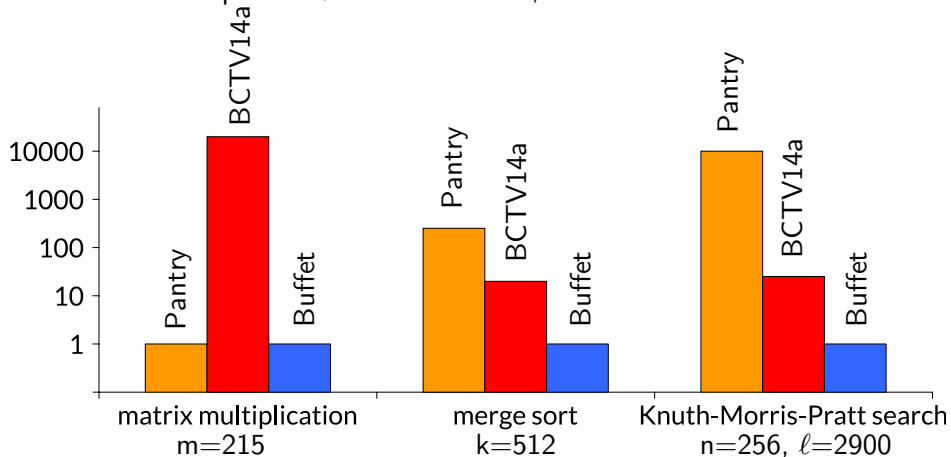
# Performance vs. expressiveness

costs	special purpose	pure	stateful	general control flow
lower	[Tha13]			
	vSQL [ZGKPP17]	Giraffe [WJBsTWW17] Allspice [VSBW13]		
	Bellman gadgetlib [BCTV14a]	Zaatar [SBVBPW13] Pinocchio [PGHR13]	xJsnark [KPS18] Geppetto [CFHKKNPZ15]	vRAM [ZGKPP18] Buffet [WSRHBW15]
	LegoSNARK [CFQ19]	Circum Ginger [SVPBBW12]	ADSNARK [BBFR15]	STARK [BBHR19]
	c0c0 [KZMQCPPsS15]	Pepper [SMBW12]	Pantry [BFRSBW13]	
				(vn)TinyRAM [BCTV14a] [BCGTV13]
				[BCTV14b] [CTV15]
higher				



## Front-end comparison

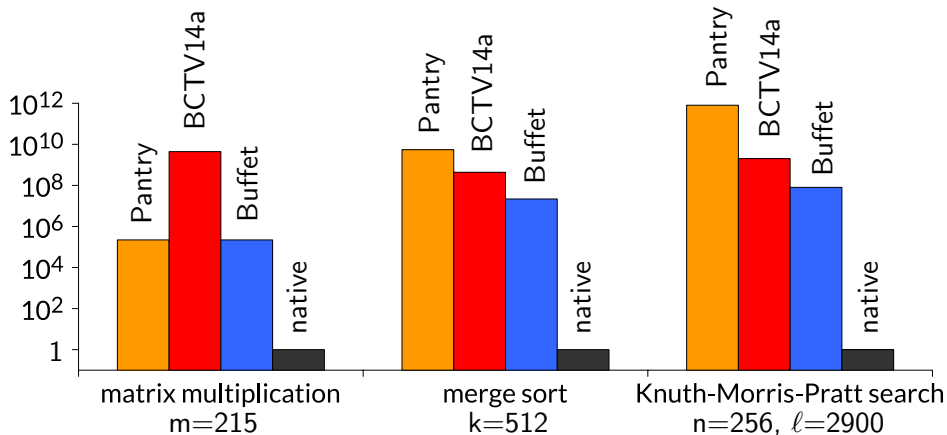
Extrapolated  $\mathcal{P}$  execution time, normalized to Buffet



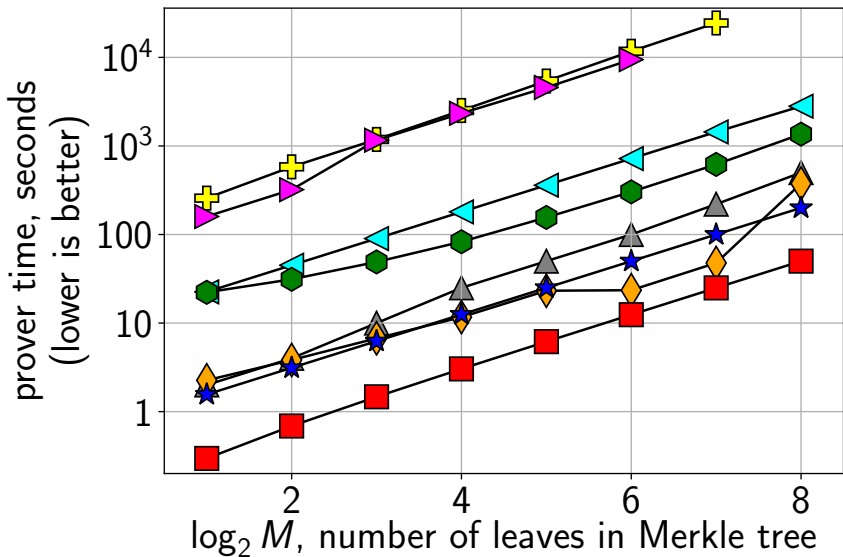
- xJsnark [KPS18] improves upon Buffet by up to  $\approx 3\times$
- vRAM [ZGKPP18] (builds on and refines [Tha13] back-end) is  $\approx 22\times$  faster than Buffet for matmult, comparable otherwise

## Reality check

Extrapolated  $\mathcal{P}$  execution time, normalized to native execution



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Numbers are from Hyrax [WTsTW18], except Libra, Aurora, and libSNARK, which are from Libra [XZZPS19]



## Reality check 2: reachable problem sizes

For  $\approx 10^7$  gates,  $\mathcal{P}$  needs  $\approx 16\text{--}32$  GiB of RAM.

Limiting computations to these sizes yields:

	Pantry	BCTV14a	Buffet
matrix multiplication $m \times m$	215	7	215
merge sort $k$ elements	8	32	512
Knuth-Morris-Pratt search needle length = $n$ haystack length = $\ell$	$n = 4,$ $\ell = 8$	$n = 16,$ $\ell = 160$	$n = 256,$ $\ell = 2900$

👉 vRAM [ZGKPP18] increases reachable sizes by  $\approx 10\times$

DIZK [WZCPS18]: distributing  $\mathcal{P}$ 's workload

Idea: run  $\mathcal{P}$  as a distributed computation

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Challenge: need to compute gigantic FFT!  
(among others)

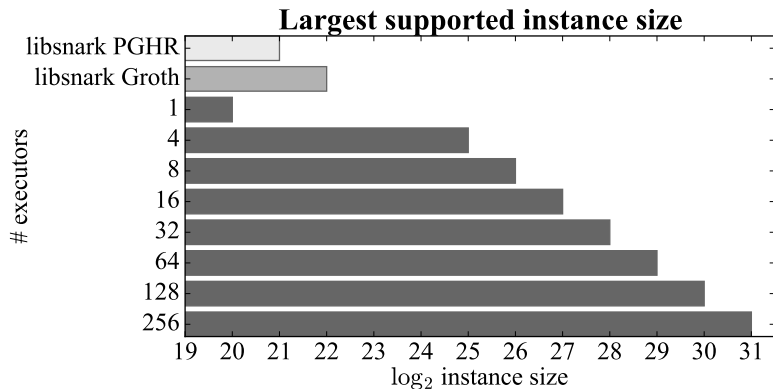
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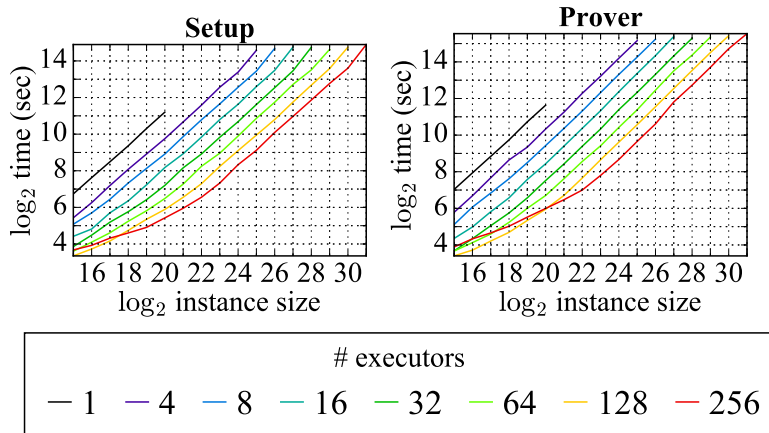
☞ [Sze11]: converts size- $n$  FFT to two  $\sqrt{n}$ -sized batches of  $\sqrt{n}$ -sized tasks

DIZK: 100× larger instances



[WZCPS18, Fig. 4]

# DIZK: 100× faster execution



[WZCPS18, Fig. 5]

## Cryptocurrencies!

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Private airdrops [BJPW19] (ePrint soon)

free money from the internet using existing credentials  
(e.g., GitHub) without revealing your identity

👉 *not* a general-purpose proof system!

Roll\_up

[https://github.com/barryWhiteHat/roll\\_up](https://github.com/barryWhiteHat/roll_up)

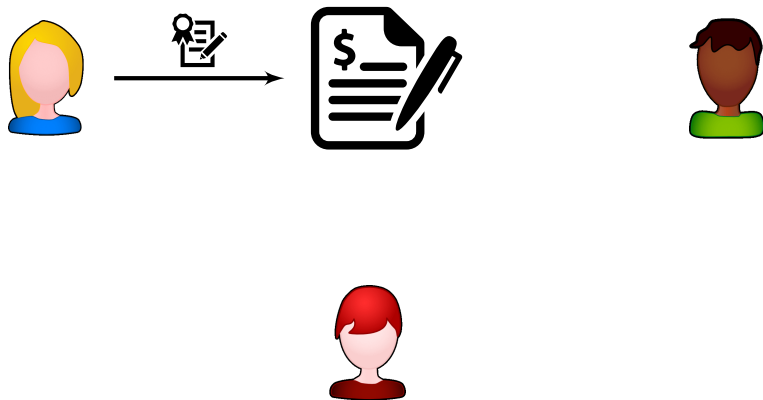
Let's build a bank out of a smart contract!



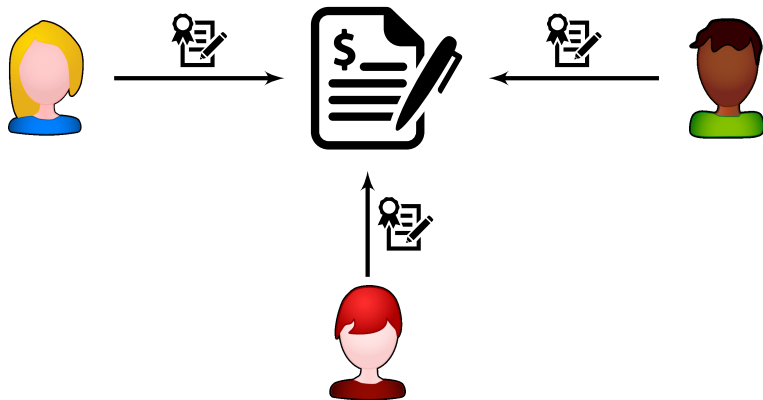
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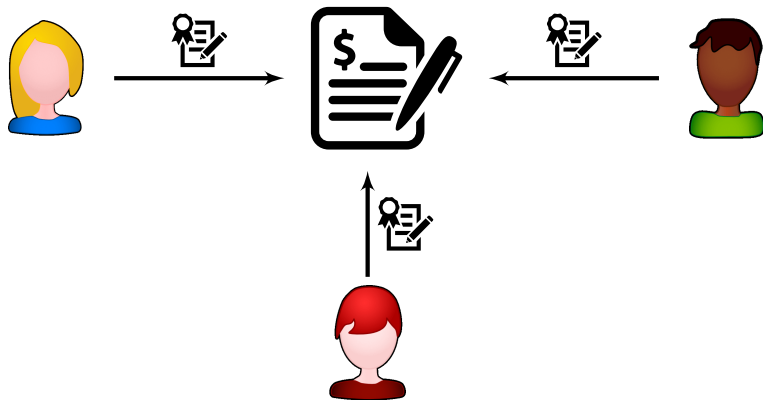
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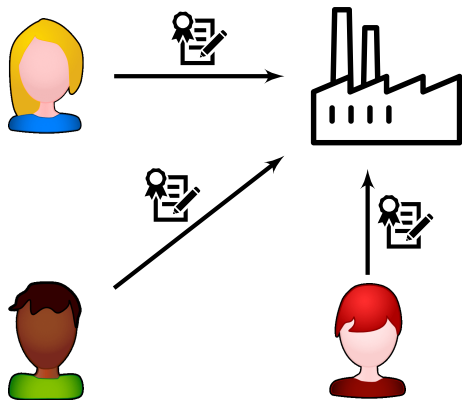


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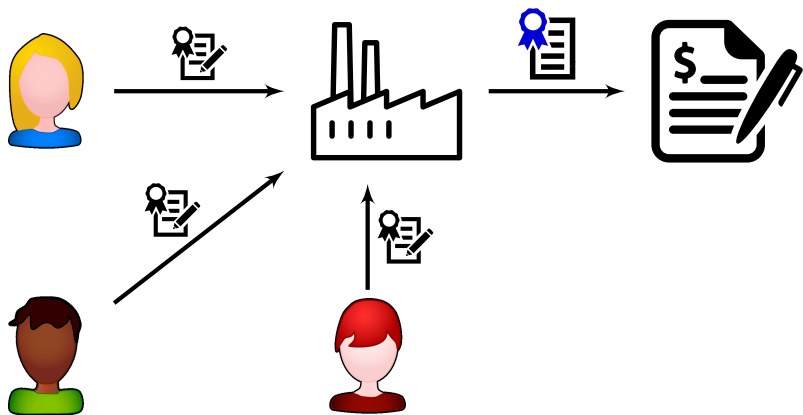
**Issues:** on-chain work and data cost \$\$\$!; slow!

Let's build a bank out of a smart contract!



**Idea:** use an off-chain, *untrusted* aggregator

Let's build a bank out of a smart contract!

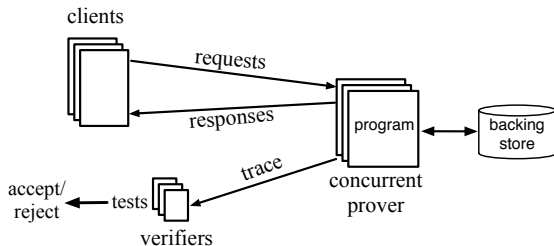


**Idea:** use an off-chain, *untrusted* aggregator to **prove** validity of a batch of transactions



# Spice [SAGL18]: verifiable *concurrent* services (in ZK)

(e.g., a cloud-hosted wallet service.)



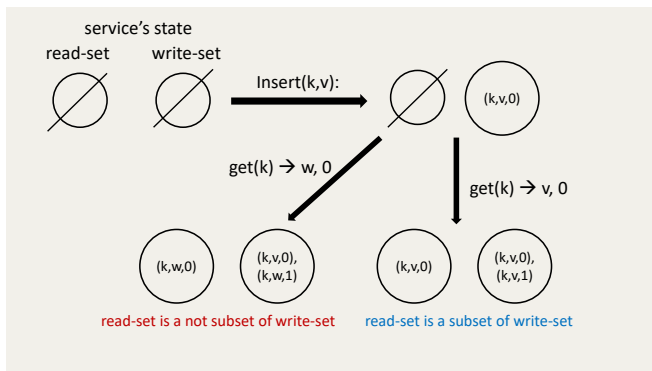
[SAGL18, Fig. 1]

**Issue:** need verifiable storage with concurrency

# Spice [SAGL18]: verifiable *concurrent* services (in ZK)

(e.g., a cloud-hosted wallet service.)

**Idea:** adapt primitives from memory checking literature [BEGKN91,CDDGE03,AEKKMPR17]



(source: Srinath's talk)

## Spice [SAGL18]: verifiable *concurrent* services (in ZK)

(e.g., a cloud-hosted wallet service.)

Performance results:

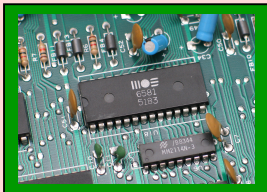
	get	put
Pantry	0.078	0.039
Pantry+Jubjub	0.153	0.076
Geppetto	0.002	0.002
Spice (1-thread)	3.6	3.6
Spice (512-threads)	1366	1370

[SAGL18, Fig. 9]

# How can we build trustworthy hardware?



## Firewall



e.g., a **custom chip** for network packet processing whose manufacture we outsource to a third party

Untrusted manufacturers can craft **hardware Trojans**



What if the chip's manufacturer inserts a **back door**?

# Untrusted manufacturers can craft hardware Trojans



What if the chip's manufacturer inserts a **back door**?

Threat: **incorrect execution** of the packet filter

(Other concerns, e.g., secret state, are important but orthogonal)

# Untrusted manufacturers can craft hardware Trojans



What if the chip's manufacturer inserts a **back door**?

The Cybercrime Economy

## Fake tech gear has infiltrated the U.S. government

by David Goldman @DavidGoldmanCNN

🕒 November 8, 2012: 3:10 PM ET

# Untrusted manufacturers can craft hardware Trojans



US DoD controls supply chain with **trusted foundries**.



## Trusted fabs are the only way to get strong guarantees

For example, stealthy trojans can thwart post-fab detection

[A2: Analog Malicious Hardware, Yang et al., Oakland16;

Stealthy Dopant-Level Trojans, Becker et al., CHES13]

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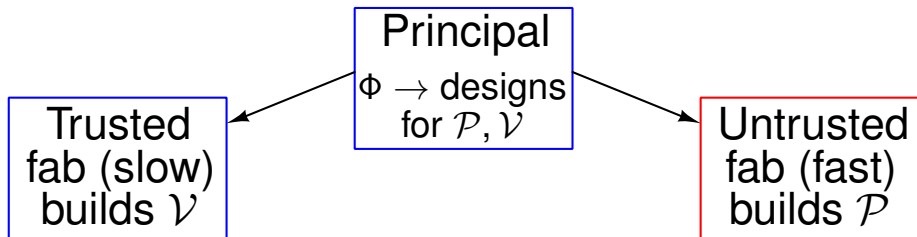
**Idea:** outsource computations to untrusted chips

## Verifiable ASICs [WHGsW16,WJBsTWW17]

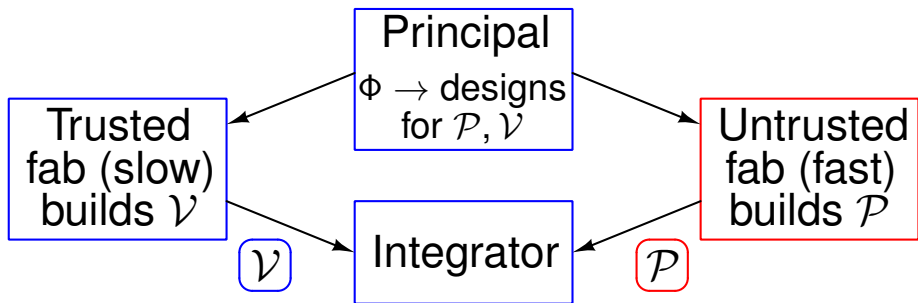
Principal

$\Phi \rightarrow$  designs  
for  $\mathcal{P}, \mathcal{V}$

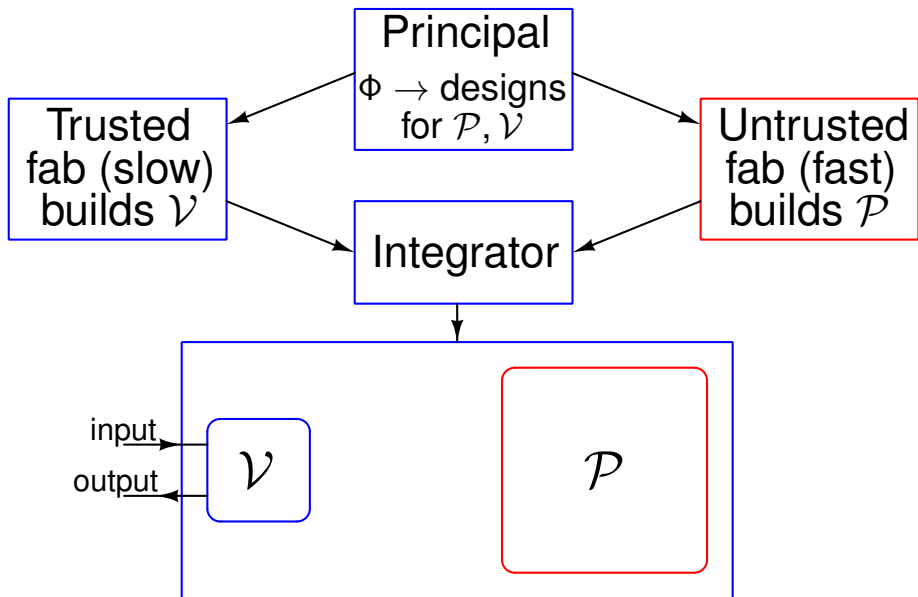
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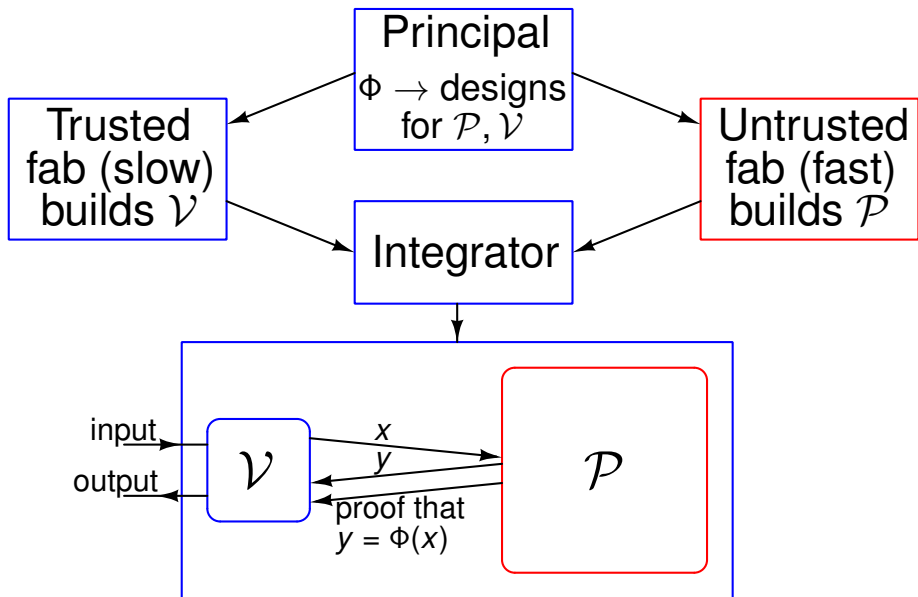


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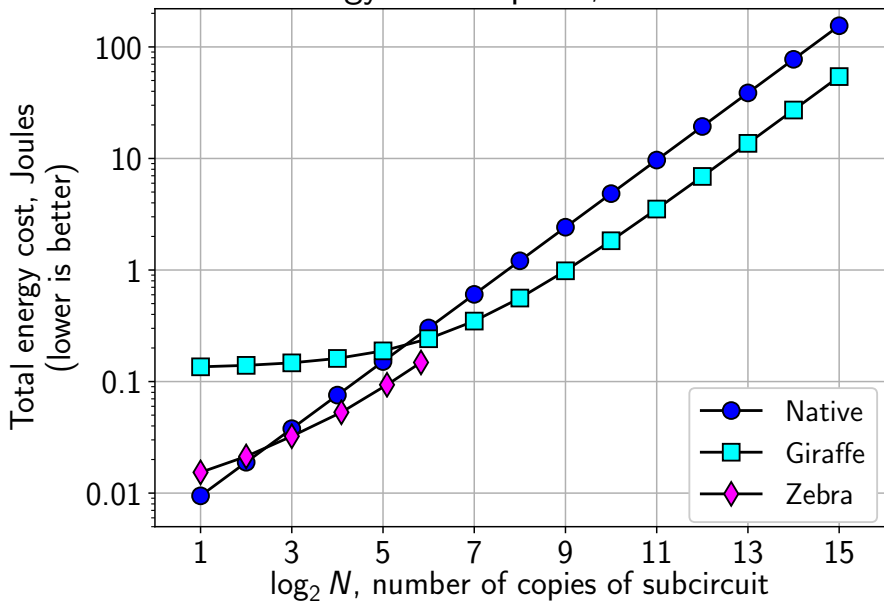


# Verifiable ASICs [WHGsW16,WJBsTWW17]



# Verifiable ASICs [WHGsW16,WJBsTWW17]: Curve25519

## Energy consumption, Joules



## Wishlist: back-ends

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memory-, communication-intensive, costly to distribute

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### updateable SRS with updateable proofs

- ➡ some steps in this direction:  
[Lip19] <https://ia.cr/2019/333>

## Wishlist: front-ends

beyond the AC model

- ☞ “natural” computations are ugly as ACs: bitwise ops, comparisons; this is a *major* cost, e.g., in SHA-256 TinyRAM [BCGTV13,BCTV14a], vRAM [ZGKPP18], STARK [BBHR19] point the way; can we go further?

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### compilers for everyone!

- 👉 recent work *hand tunes* statements, relies on authors' intuition and implicit knowledge
  - let's *systematize* this knowledge, *automate* tuning
- ✓ improved accessibility and real-world deployability
- ✓ highly leveraged work for the research community: simpler, higher quality evaluations, easier-to-interpret results



## Recap

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